Statistical Learning on Metric Spaces

Hông Vân Lê

Institute of Mathematics, Czech Academy of Sciences
Zitna 25, 11567 Praha 1, Czechia.
hvle@math.cas.cz

In statistical learning theory we design models for estimation and approximation of stochastic functional dependencies between empirical data [1], [5], [6], [9], [10]. In classical learning theory, the generalization ability of a learning model can be formulated in terms of concentration-of-measure inequalities. In my talk, I shall propose categorical and geometric methods, developing methods in [4], [7], [8], and utilizing the convergence in outer probability, which has been employed in the study of empirical processes [3], for proving the generalization ability of supervised learning models on Polish subspaces of $\mathbf{R}^n$. Our new results make precise and extend previous works due to Cucker-Smale [1] and Vapnik [9], which concern a class of supervised learning models. I shall discuss the relation of the obtained results with the open problem of the generalization ability of neural networks, which has been raised and discussed in [2]. A part of my talk is based on my e-print [5].

References

[1] F. Cucker and S. Smale, On mathematical foundations of learning, Bulletin of AMS, 39 (2002), 1-49.

[2] J. Berner, P. Grohs, G. Kutyniok, P. Petersen, The Modern Mathematics of Deep Learning, in: Mathematical Aspects of Deep Learning, Edited by P. Grohs, G. Kutyniok, Cambridge University Press 2022, 1-111, arXiv:2105.04026.

[3] R. M. Dudley, Uniform Central Limit Theorems, Second Edition, Cambridge, 2014.

[4] J. Jost, H. V. Lê, and T. D. Tran, Probabilistic morphisms and Bayesian nonparametrics, Eur. Phys. J. Plus 136, 441 (2021), arXiv:1905.11448.

[5] H. V. Lê, Supervised learning with probabilistic morphisms and kernel mean embeddings, arXiv:2305.06348.

[6] H. V. Lê, H. Q. Minh, F. Protin, W. Tuschmann, Mathematical Foundations of Machine Learning, in preparation, to be published by Springer Nature.

[7] B. Sriperumbudur, A. Gretton, K. Fukumizu, B. Schölkopf, and G.R.G. Lanckriet, Hilbert space embeddings and metrics on probability measures, J. Math. Learn. Res. 11, 1517-1561, (2010).

[8] B. Sriperumbudur, On the optimal estimation of probability measures in weak and strong topologies, Bernoulli, 22(3):1839-1893, 08, 2016.

[9] V. Vapnik, Statistical Learning Theory. John Willey & Sons, 1998.

[10] V. Vapnik, The Nature of Statistical Learning Theory. Springer, 2nd Edition, 2000.