Almost rigidity for Green function with nonnegative Ricci curvature

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Let (M^n, g) be a Riemannian manifold of dimension $n \geq 3$ with nonnegative Ricci curvature. Assume that (M^n, g) is nonparabolic, namely

$$\int_0^\infty \frac{s}{\mathrm{vol} B_s(x)} d\mathrm{vol} < \infty. \tag{1}$$

Then the positive Green function $G^{M^n}(x, y)$ is defined by

$$G^{M^n}(x, y) := \int_0^\infty p^{M^n}(x, y, t)dt, \tag{2}$$

where p^{M^n} denotes the heat kernel. This allows us to define the smoothed distance function $b_x^{M^n}$ from x as follows;

$$b_x^{M^n} := \left(G^{M^n}(x, \cdot)\right)^{\frac{1}{2-n}}. \tag{3}$$

In the Euclidean case, we know

$$G^{\mathbb{R}^n}(x, y) = \frac{|x-y|^{2-n}}{n(n-2)\omega_n}, \tag{4}$$

thus $b_x^{\mathbb{R}^n}$ coincides with the distance function from x up to multiplying a dimensional constant $(n(n-2)\omega_n)^{\frac{1}{n-2}}$.

Colding proved

$$|\nabla b_x^{M^n}|(y) \leq (n(n-2)\omega_n)^{\frac{1}{n-2}}, \quad \forall y \in M^n \setminus \{x\}. \tag{5}$$

Moreover the equality holds for some $y \in M^n \setminus \{x\}$ if and only if the manifold is Euclidean. This is called a rigidity result.

In this talk we provide the almost rigidity result which states naively that if the left-hand-side of (5) at some point y is close to the right-hand-side, then the manifold is close to the Euclidean space in some sense. In order to do so, we use a nonsmooth geometric analysis. This is a joint work with Yuanlin Peng (Tohoku University).