Impossibility of quasisymmetric Gaussian uniformization, via decay rates of harmonic functions, for Brownian motion on some planar Sierpiński carpets

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It is an established result in the field of analysis of heat equations and associated diffusions (Markov processes with continuous sample paths) on fractals, that the heat kernel (the transition density of the diffusion) typically satisfies analogs of Gaussian bounds which involve a space-time scaling exponent β greater than two and thereby are called SUB-Gaussian bounds. The exponent β, called the walk dimension of the diffusion, could be considered as representing "how close the geometry of the fractal is to being smooth". It has been observed by Kigami in [Math. Ann. **340** (2008), 781–804] that, in the case of the standard two-dimensional Sierpiński gasket, one can decrease this exponent to two (so that Gaussian bounds hold) by suitable changes of the metric and the measure while keeping the associated Dirichlet form (the quadratic energy functional) the same. Then it is natural to ask how general this phenomenon is for diffusions on fractals.

In fact, it turns out that the above phenomenon, that one can decrease the exponent β to two so that Gaussian bounds hold, seems to happen only for a very limited class of self-similar fractals. This talk is aimed at presenting the result that this phenomenon indeed does NOT happen for the Brownian motion on a class of two-dimensional Sierpiński carpets, as well as for the Brownian motion on the standard three- and higher-dimensional Sierpiński gaskets. The key to the proof is some knowledge about decay rates of harmonic functions, which for Sierpiński carpets seems new and is of independent interest.

This talk is based on joint works with Mathav Murugan (University of British Columbia). The results for planar Sierpiński carpets is in progress, and that for the standard higher-dimensional Sierpiński gaskets is given in [Invent. math. **231** (2023), 263–405].