Geometry of weighted Finsler spacetimes

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This talk will be a review of the theory of weighted Lorentz–Finsler manifolds. A Lorentz–Finsler manifold is a generalization of a Lorentzian manifold in the same way that a Finsler manifold generalizes a Riemannian manifold. One can further equip a Lorentz–Finsler manifold with a time orientation as well as a weight, then we have a weighted Finsler spacetime. In this general framework, we can successfully develop the theory of Ricci curvature (singularity theorems, various comparison theorems, curvature-dimension condition, etc.). Especially, the timelike lower Ricci curvature bound $\mathrm{Ric}_N \geq K$ turns out equivalent to the timelike curvature-dimension condition $\mathrm{TCD}_q(K,N)$ for all $K \in \mathbb{R}$ and $N \in (-\infty, 0] \cup [n, +\infty]$ with sharp coefficients, extending and improving McCann and Mondino–Suhr's characterizations in the Lorentzian setting. This talk is based on joint works with Mathias Braun, Yufeng Lu and Ettore Minguzzi.

References

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