Isoperimetric inequalities vs. upper curvature bounds

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Upper curvature bounds in the sense of Alexandrov are synthetic generalizations of upper sectional curvature bounds in Riemannian geometry. Typically, they are expressed in terms of triangle comparison, serving to control divergence of geodesics and consequently entailing related angle constraints. In contrast, isoperimetric inequalities establish a connection between area and length. More precisely, an isoperimetric inequality provides an upper bound for the area of a least area disc in terms of its boundary length. The associated Dehn function $D(r)$ is defined as the largest area of a least area disc with boundary length at most r. For simply connected Riemannian manifolds of constant curvature k the Dehn function $D_k(r)$ is explicitly computable. A recent fundamental result of Lytchak-Wenger states that a locally compact length metric space has curvature at most k iff its Dehn function is bounded above by $D_k(r)$. In the talk I will discuss this result and explain how to drop the local compactness assumption.
Joint work with Stefan Wenger.